

Subgroups

A subgroup of lower order can be formed from a larger group

Group of order = 8 $G_8^{(2)}$

$G_8^{(2)}$	E	A	B	C	D	F	G	H
E	E	A	B	C	D	F	G	H
A	A	B	C	E	G	H	F	D
B	B	C	E	A	F	D	H	G
C	C	E	A	B	H	G	D	F
D	D	H	F	G	E	B	C	A
F	F	G	D	H	B	E	A	C
G	G	D	H	F	A	C	E	B
H	H	F	G	D	C	A	B	E

The order of any subgroup, g , of a group of order h must be a divisor of h

$$\frac{h}{g} = k, \text{ where } k \text{ is an integer}$$

For the group $G_8^{(2)}$ the subgroups are:

$$g = 1 \quad \{E\}$$

$$g = 2 \quad \{E, B\} \quad \{E, D\} \quad \{E, F\} \quad \{E, G\} \quad \{E, H\}$$

$$g = 4 \quad \{E, A, B, C\} \quad \{E, B, D, F\} \quad \{E, B, G, H\}$$

Inverses

$$\begin{aligned}
 E^{-1} &= E \\
 A^{-1} &= C \\
 B &= B \\
 C^{-1} &= A \\
 D^{-1} &= D \\
 F^{-1} &= F \\
 G^{-1} &= G \\
 H^{-1} &= H
 \end{aligned}$$

Similarity transforms for E:

$$\begin{aligned}
 E^{-1}EE &= E \\
 A^{-1}EA &= CEA = E \\
 B^{-1}EB &= BEB = E \\
 C^{-1}E &= AEC = E \\
 DED &= E \\
 FEF &= E \\
 GEG &= E \\
 HEH &= E
 \end{aligned}$$

Class of order 1: $\{E\}$ For A : $\{A, C\}$

$$\begin{aligned}
 E^{-1}AE &= A \\
 A^{-1}AA &= CAA = CB = A \\
 B^{-1}AB &= BC = A \\
 C^{-1}AC &= AAC = AE = A \\
 DAD &= DG = C \\
 FAF &= FH = C \\
 GAG &= GF = C \\
 HAH &= HD = C
 \end{aligned}$$

For B : $\{B\}$

$$\begin{aligned}
 EBE &= B \\
 C^{-1}BA &= CC = B \\
 B^{-1}BB &= BE = B \\
 A^{-1}BC &= AA = B \\
 D^{-1}BD &= DF = B \\
 F^{-1}BF &= FD = B \\
 G^{-1}BG &= GH = B \\
 H^{-1}BH &= HQ = B
 \end{aligned}$$

For D : $\{D, F\}$

$$\begin{aligned}
 EDE &= D \\
 CDA &= CH = F \\
 BDB &= BF = D \\
 ADC &= AG = F \\
 DDD &= DE = D \\
 FDF &= FB = D \\
 GDG &= GC = F \\
 HDH &= HA = F
 \end{aligned}$$

For G : $\{G, H\}$

$$\begin{aligned}
 EGE &= G \\
 CGA &= CD = H \\
 BGB &= BH = G \\
 AGC &= AF = H \\
 DGD &= DA = H \\
 FGF &= FC = H \\
 GGG &= G \\
 HGH &= HB = G
 \end{aligned}$$

Classes : order 1 : $\{E\}$ and $\{B\}$ order 2 : $\{A, C\}$, $\{D, F\}$, and $\{G, H\}$

no classes of order 4