Subgroups

A subgroup of lower order can be formed from a larger group.

Group of order 8, \( G_8^{(2)} \)

<table>
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<tr>
<th>( G_8^{(2)} )</th>
<th>E</th>
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<th>C</th>
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The order of any subgroup, \( g \), of a group of order \( h \) must be a divisor of \( h \):

\[
\frac{h}{g} = k, \text{ where } k \text{ is an integer}
\]

For the group \( G_8^{(2)} \), the subgroups are:

- \( g = 1 \) \( \{E\} \)
- \( g = 2 \) \( \{E, B\} \) \( \{E, D\} \) \( \{E, F\} \) \( \{E, G\} \) \( \{E, H\} \)
- \( g = 4 \) \( \{E, A, B, C\} \) \( \{E, B, D, F\} \) \( \{E, B, G, H\} \)
Inverses

\[ E^{-1} = E \]
\[ A^{-1} = C \]
\[ B = B \]
\[ C^{-1} = A \]
\[ D^{-1} = D \]
\[ F^{-1} = F \]
\[ G^{-1} = G \]
\[ H^{-1} = H \]

Similarity transforms for \( E \):

\[ E^{-1}EE = E \]
\[ A^{-1}EA = CE\!A = E \]
\[ B^{-1}EB = B\!E\!B = E \]
\[ C^{-1}E = A\!E\!C = E \]
\[ DED = E \]
\[ FEF = E \]
\[ GEG = E \]
\[ HEH = E \]

Class of order 1: \( \{ E \} \)

For \( A \): \( \{ A, C \} \)

\[ E^{-1}AE = A \]
\[ A^{-1}AA = CAA = CB = A \]
\[ B^{-1}AB = BC = A \]
\[ C^{-1}AC = AAC = AE = A \]
\[ DAD = DG = C \]
\[ FAF = FH = C \]
\[ GAG = GF = C \]
\[ HAH = HD = C \]

For \( B \): \( \{ B \} \)

\[ EB\!E = B \]
\[ C\!B\!A = CC = B \]
\[ B\!B\!B = BE = B \]
\[ A\!B\!C = AA = B \]
\[ D\!B\!D = DF = B \]
\[ FBF = FD = B \]
\[ G\!B\!G = GH = B \]
\[ H\!BH = HG = B \]

For \( D \): \( \{ D, F \} \)

\[ EDE = D \]
\[ CDA = CH = F \]
\[ BDB = BF = D \]
\[ ADC = AG = F \]
\[ DDD = DE = D \]
\[ FDF = FB = D \]
\[ GDG = GC = F \]
\[ HDH = HA = F \]

For \( G \): \( \{ G, H \} \)

\[ EG\!E = G \]
\[ CGA = CD = H \]
\[ BGB = BH = G \]
\[ AGC = AF = H \]
\[ DGD = DA = H \]
\[ FGF = FC = H \]
\[ GGG = G \]
\[ HGH = HB = G \]

Classes:

order 1: \( \{ E \} \) and \( \{ B \} \)

order 2: \( \{ A, C \}, \{ D, F \}, \) and \( \{ G, H \} \)

no classes of order 4.