

# Equilateral Triangle Representations

|                                |  |   |   |   |   |   |
|--------------------------------|--|---|---|---|---|---|
| $\Gamma^1$                     | 1  | 1   | 1   | 1   | 1   | 1   |
| identity                       |  |   |   |   |   |   |
| $\Gamma^2$                     | 1  | -1  | -1  | -1  | 1   | 1   |
| determinant                    |  |   |   |   |   |   |
| $\Gamma^3$                     | $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ | $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ | $\begin{bmatrix} -1/2 & \sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{bmatrix}$ | $\begin{bmatrix} -1/2 & -\sqrt{3}/2 \\ -\sqrt{3}/2 & 1/2 \end{bmatrix}$ | $\begin{bmatrix} -1/2 & \sqrt{3}/2 \\ -\sqrt{3}/2 & -1/2 \end{bmatrix}$ | $\begin{bmatrix} -1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & -1/2 \end{bmatrix}$ |
| matrices                       |  |   |   |   |   |   |
| Element<br>(original<br>group) | E  | A   | B   | C   | D   | F   |

$\Gamma^1$  and  $\Gamma^2 \Rightarrow$  Homomorphic

$\Gamma^3 \Rightarrow$  isomorphic and homomorphic

In vector form

|                 |   |    |                      |                       |                       |                       |   |
|-----------------|---|----|----------------------|-----------------------|-----------------------|-----------------------|---|
|                 | E | A  | B                    | C                     | D                     | F                     |   |
| $\Gamma^1$      | 1 | 1  | 1                    | 1                     | 1                     | 1                     |   |
| $\Gamma^2$      | 1 | -1 | -1                   | -1                    | 1                     | 1                     |   |
| $\Gamma^3$      | 1 | 1  | $-\frac{1}{2}$       | $-\frac{1}{2}$        | $-\frac{1}{2}$        | $-\frac{1}{2}$        | } positions<br>$\begin{bmatrix} 1,1 & 1,2 \\ 2,1 & 2,2 \end{bmatrix}$ |
| $\Gamma^3_{11}$ |   |    |                      |                       |                       |                       |   |
| $\Gamma^3_{12}$ | 0 | 0  | $\frac{\sqrt{3}}{2}$ | $-\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{3}}{2}$  | $-\frac{\sqrt{3}}{2}$ |   |
| $\Gamma^3_{21}$ | 0 | 0  | $\frac{\sqrt{3}}{2}$ | $-\frac{\sqrt{3}}{2}$ | $-\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{3}}{2}$  |   |
| $\Gamma^3_{22}$ | 1 | -1 | $\frac{1}{2}$        | $\frac{1}{2}$         | $-\frac{1}{2}$        | $-\frac{1}{2}$        |   |